NZSEC 2024

Team # 1185 Report

"How many airships would be required to replace the Cook Strait Ferries?"

1 Abstract

The purpose of this report is to calculate the number of airships required to replace the Cook Straight Ferries in New Zealand. We have decided that Zeppelins is the primary airship that will be used in order to travel across the Cook Straight from the North island to the South island. Zeppelins provided the most practical airship that would ensure the viability of replacing the efficiency, effectiveness, and sustainability of the Cook Strait Ferries. Through mathematical modelling, we have created a fleet of 10 225m long Zeppelins that are able to transport the peak amount of people, cars, and cargo at any given time. They will operate for passengers and cars from 8am - 6pm each day and cargo during the night. This will equate to around 64 one way trips per day to accommodate the current maximum capacity of the Cook Strait Ferries.

2 Introduction

One of the integral movement pathways in New Zealand is the connection between the North and South Island. Hence, the Cook Strait Ferries have a critical role in the movement of passengers, vehicles, and goods across. This transportation link is not only vital for the local economy and daily life of New Zealanders but also for the country's tourism industry and supply chains. As technology continues to develop, transportation must also evolve at the same rate. Therefore it is vital to explore alternative methods that can replace or work in tandem with current systems, while ensuring a stability through efficiency, reliability, and sustainability.

Recent news articles have stated "Satisfaction levels low with Cook Strait Ferries".[1] Hence with satisfaction levels low as a result of risk concerns, delay times, and availability, alternatives for the transport between the islands must be considered.

In this report, we will mathematically model how many Zeppelin airships that will be needed in order to have a sufficient substitute for the current maximum passengers, cars, trucks, and goods that need to be transported across the Cook Strait.

[1] https://www.stuff.co.nz/nz-news/350373016/satisfaction-levels-low-cook-strait-ferries-some-aratere-repairs-complete-after

3 Interpretation and Definition of Key Terms:

In this report, we define airships as an aircraft that is kept buoyant by a body of gas and is power driven. Specifically, rigid Zeppelins will be used as the primary airship for transport of goods and passengers.

In addition, ”replacement” refers to substituting the existing Cook Strait Ferries with Zeppelins to provide the same level of transportation service between the North and South Islands of New Zealand. This involves modelling whether Zeppelins can meet or exceed the transport capacity, efficiency, and reliability provided by the current ferries.

We define Cook Strait Ferries as a transport pathway between the North and South islands across the Cook strait that has the ability to transport passengers, cars, trucks, and goods at regular intervals throughout the day.

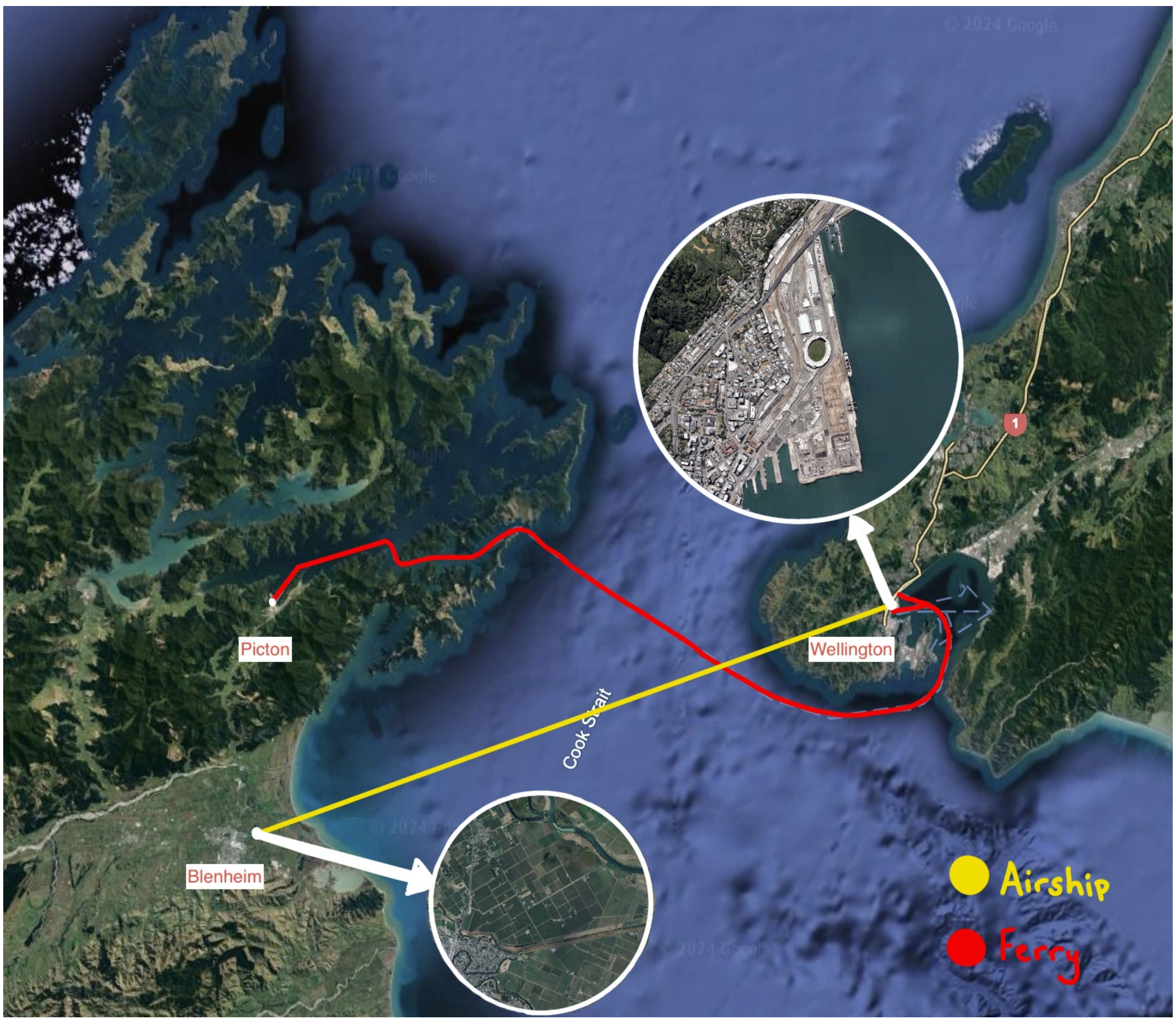
4 Assumptions

- We do not believe that cost is within the scope of this question, so we will be assuming that there is enough money to purchase or construct any number of airships, and to run all of these airships, without issue.
- We will also assume that the physical size and shape of cargo stays relatively constant in relation to weight, since weight is the limiting factor for carrying cargo, due to the very large size of the lifting envelope.
- Additionally we will assume that the introduction of airships does not shift the amount of passenger interest on travel mode (plane, boat, airship) i.e. the airships will only have to carry the same number that the ferries did, and there are no additional people drawn in by shorter travel times or any other factors.
- We assume that our Zeppelins have similar general proportions to the Hindenburg, with the material upgrades of modern Zeppelins, such as the Zeppelin NT. This places the length of our theoretical Zeppelin at somewhere between 245 metres, which was the length of the Hindenburg[2], and 75 metres, the length of the Zeppelin NT[3]. Width is proportional to length, so any length within these bounds will also have a reasonable width. Any hypothetical Zeppelins which have lengths (and widths) outside of this range will have inaccurate lift and weight calculations, as our model is based on these 2 airships.
- When we need to do calculations involving atmospheric conditions, we used the following set of normal values: 1 bar (100,000Pa) in atmospheric pressure, and 273.15 K in temperature.

[2]https://www.airships.net/hindenburg/size-speed/
[3]https://zeppelinflug.de/en/faqs

5 Location

The flight model that we have provided is a round trip from Wellington to Blenheim. The question suggests that the Cook Strait Ferries that need to be replaced has the basic function of transporting people, transport modes, and goods across the strait and between the North and South islands. Hence the most direct and feasible route will be from Wellington to Blenheim. This is because of the vast amounts of open flat land that is available for storage, landing, and refueling. Compared to Picton which is wedged in a valley of mountain ranges. The amount of distance needed to travel from Wellington to Picton via the Interislander is 100km, however this can be reduced to 65km by flying in a Zeppelin to Blenheim.



6 Modelling

6.1 Introduction

In order for a fleet of airships to replace the Cook Strait Ferries, the following inequality must be satisfied:

$$n \times t \times c \geq W$$

n is the number of airships in the fleet, t is the number of trips each airship can do in a day, c is the amount of weight each airship can carry and W is the maximum total weight of the freight and passengers that will have to be transported across the Cook Strait on a single day. To answer "How many airships would be required to replace the Cook Strait Ferries?", we just need to find t, c and W and then solve for n.

6.2 Weight Calculations

We can model a Zeppelin as a prolate spheroid, with a minor axis of 2a, and a major axis of 2c. By the ideal gas law, we can estimate the air it displaces, and hence how much it is able to lift.

$$\begin{aligned} PV &= nRT \\ n &= \frac{PV}{RT} \end{aligned}$$

With:

$$\begin{aligned} V &= \frac{4\pi}{3}a^2c \\ P &= 100,000\text{Pa} \\ T &= 273.15\text{K} \end{aligned}$$

The Molar Mass of air is 0.02896 kilograms per mole, so the total weight of the air displayed by our Zeppelin is:

$$\begin{aligned} m &= \frac{0.02896\frac{4\pi}{3}a^2c \times 100,000}{273.15 \times 8.314} \\ &= 5.341656a^2c \end{aligned}$$

This yields the theoretical maximal lift. We will subtract the structural weight, the fuel weight, and miscellaneous weights to obtain the effective available lifting capacity.

$$L_e = L_(max) - S - F - M$$

6.3 Structural Weight

The structural weight is composed of the lifting gas, the internal supports, and the envelope.

$$S = m_g + m_i + m_e$$

6.3.1 Gas

Due to the dangers of hydrogen, we will use helium as the only viable alternative as a lifting gas. To calculate the mass of helium, we will multiply the volume of the Zeppelin by the density of pure helium.

$$m_g = \frac{4\pi}{3} a^2 c \times 0.166$$

6.3.2 Internal Supports

The Hindenburg had 15 internal supports, each composed of 3 circular rings of aluminium[4]. Each of these supports is 0.1 metres wide, and 0.03 metres thick. Our supports will be made of mylar[5], used in the Zeppelin NT, which has a density of 1390 kgm^{-}3. Assuming that the spacing of rings is constant for different sized Zeppelins, the equation for the mass of internal supports is:

$$m_i = \frac{6c}{10} 2\pi a \times 0.1 \times 0.03$$

6.3.3 Envelope

The volume of the envelope is equal to the surface area of the spheroid, multiplied by its thickness. Multiplying this by the density of mylar, the material the Zeppelin NT is composed of, will then give mass:

$$\begin{aligned} m_e &= A * T * d \\ m_e &= 1390 \times 0.0015 \times 2\pi(a^2 + \frac{ac}{e} \arcsin e) \\ &= 13.1004(a^2 + \frac{ac}{e} \arcsin e) \end{aligned}$$

Where $e = \sqrt{1 - \frac{a^2}{c^2}}$.

[4]https://www.airships.net/hindenburg/hindenburg-design-technology/
[5]https://www.airships.net/zeppelin-nt/

6.3.4 Model Testing

We will input the data from the Hindenburg in order to test the accuracy of our buoyancy model.

$$L_e(20.6, 123) = 150,643\text{kg}$$

The Hindenburg had 215,910 kg total lifting power, and 118,000 kg of structural weight, which gives 97,910 kg of effective lifting power[6]. This is similar to the value for the Hindenburg, and hence this indicates our buoyancy model is somewhat accurate. The discrepancy is likely due to our usage of modern materials when calculating the values for the envelope and the structural frame.

6.4 Other weight

6.4.1 Fuel

A Zeppelin moving at cruising speed will consume 15 gallons of aviation fuel per hour[7]. The theoretical maximum amount of time one of our Zeppelins will be operating is 12 hours. For safety, we will require twice the bare minimum amount of fuel to make this journey to be present on board. Multiplying these values, and converting the units, gives 1000 kg of fuel to be on board each Zeppelin. We don't need to consider the weight of the Zeppelin, as lift is generated by the helium gas, and we assume that drag is negligible.

6.4.2 Miscellaneous Weights

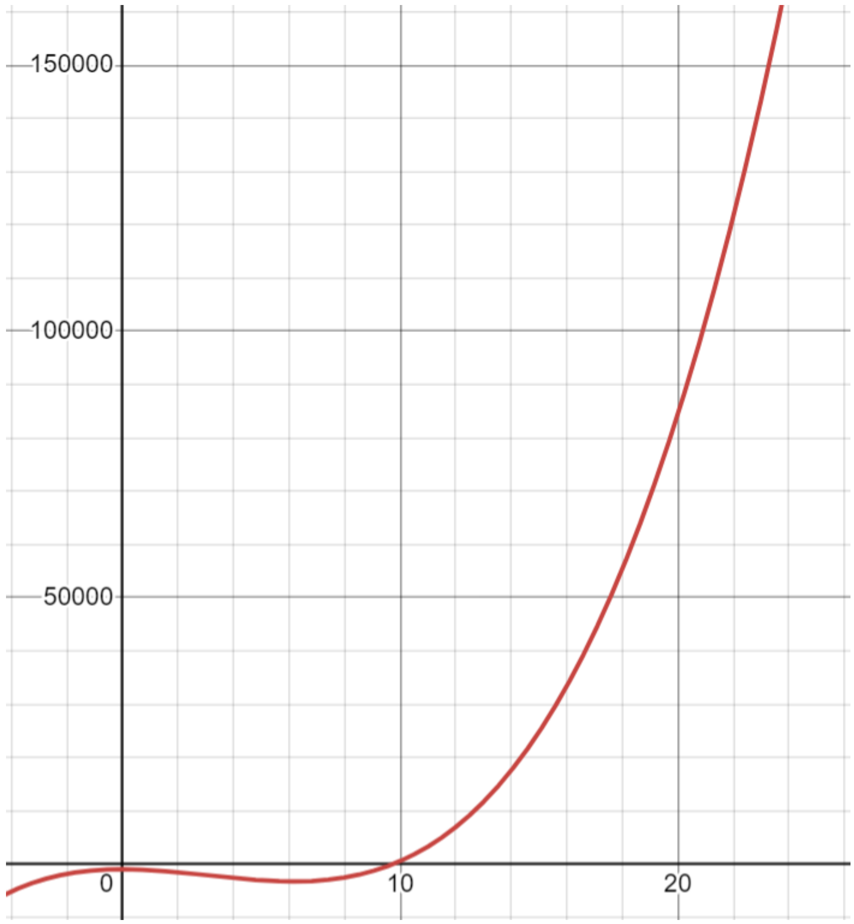
Miscellaneous weights account for other non-passenger weights. We have used the data for the Hindenburg, which includes oil, ballast, and interior furnishings[8]. We assume that this is proportional to the volume of the Zeppelin. Calculating using Hindenburg data yields the following expression for miscellaneous weights.

$$m_m = 0.1617V$$

With $V = \frac{4\pi}{3}a^2c$

[6]https://www.airships.net/hindenburg/size-speed/
[7]https://www.dailynews.com/20080519/blimps-surprisingly-fuel-efficient/
[8]https://www.airships.net/hindenburg/

6.5 Graphical Summary



The x-axis is a in metres, and the y-axis is the weight the Zeppelin can carry in kg.

We assume that our Zeppelins will have scaled dimensions of the Hindenburg. Thus, we can replace c with $\frac{123a}{20.6}$. This gives us an equation for the effective lifting capacity of a hypothetical Zeppelin in one variable. Therefore, we can plot the L_e for a variety of different Zeppelins:

6.6 Total Weight Calculations

6.6.1 Number of passengers and vehicles

According to Interislander[9] "Each year, the Interislander makes 3,500 trips and carries 825,000 passengers, 52,000 rail wagons, 21,000 trucks and 250,000 cars." However, these figures don't include Bluebridge, and we were unable to find similar data for them. According to a 2021 report from the Ministry of Transport[10] "Normally 1.2 million passengers travel on the Cook Strait ferries each year, with this projected to grow to 1.7 million within 20 years." We can use these figures to extrapolate the data for Bluebridge, by assuming that the ratio of passengers from Interislander to Bluebridge is the same as the ratios for trucks and cars. This won't apply to rail wagons, because Aratere, which is operated by Interislander, is New Zealand's only rail ferry[11]. This gives us the following total yearly figures:

[9] https://www.interislander.co.nz/explore/the-history-of-the-interislander-ferry

[10] https://www.transport.govt.nz/assets/Uploads/11_Detailed-Business-Case_Interislander-Ferries-and-Terminals.pdf

[11] https://en.wikipedia.org/wiki/DEV_Aratere

- 1,200,000 passengers
- 363,636 cars
- 30,545 trucks
- 52,000 rail wagons

6.6.2 Peak daily number

We now need to find the peak daily numbers of passengers and vehicles. This Zeppelin company[12] reports that passenger flights are only operated up to wind speeds of 35km/h. By extrapolating data from Niwa, we estimate that Zeppelins will only be able to operate on roughly 80% of days. From this we can find the average daily number of passengers and vehicles. We assume that the peak number of passengers, cars and trucks will be double this average number. For rail wagons we assume that the peak number will be roughly the same as the average, since they can be made to follow strict timetables. This gives us the following peak daily numbers:

- 8220 passengers
- 2491 cars
- 209 trucks
- 178 rail wagons

6.6.3 Weight of passengers and vehicles

For the passengers we assume an average weight of 80kg, plus 10kg of luggage and 15kg for the seat, giving us an average of 105kg per person. However, we will use 130kg to give us a margin of error. The average weight of a fully-laden passenger car in New Zealand is roughly 2000kg[13]. For trucks, we use an average of 12,000kg. This gives us the following weights:

- 1,068,600kg of passengers
- 4,982,000kg of cars
- 2,508,000kg of trucks

Giving us a total W value of 8,558,600kg for passengers, cars and trucks. As part of our proposal, rail wagons will be transported at night, so most of the weight will be transported during the day. As such this is the W value that we need to satisfy.

[12] https://zeppelinflug.de/en/faqs
[13] https://interactives.stuff.co.nz/2021/09/heavy-traffic-nz-vehicle-weight/

6.7 Finding n

We have decided to use Zeppelins with a width of 45m and a length of 225m, which is slightly smaller than the Hindenburg. According to our model, this gives us a c value 132,278kg. Going back to our inequality value from the beginning, this becomes:

$$n \times t \times 132,278\text{kg} \geq 8,558,600\text{kg}$$
$$n \times t \geq 65$$

This can easily be accomplished with $n = 10$ and $t = 7$. The distance from Blenheim to Wellington is roughly 65km and the cruising speed of Zeppelins is roughly 70km/h[14]. Taking into account loading and unloading, a one-way trip should take roughly 1 hour and a half, so a Zeppelin should be able to complete 7 one-way trips in half a day, and therefore the rail cargo which is lighter can also be transported in the other half of the day.

6.7.1 Our Model

Our model states that a fleet of 10 Zeppelins will be able to match the amount of cargo and goods being transported per day. They will fly for passengers during the day from 8am-6pm and will transport the cargo on rail wagons overnight. Cars will be matched with the passengers to ensure everything gets there in tandem. This will result in roughly 65 one way flights per day at its peak and can carry up to 8220 passengers, 2491 cars and 209 trucks.

7 Conclusion

In this paper, we have calculated the number of airships required to replace the current ferry services operating across the Cook Strait. Through mathematical modelling, it has been calculated that a fleet of 10 (225m long, 45m wide) Zeppelins is sufficient in replacing the amount of passengers, vehicles, and cargo that are transported by ferry currently.

8 Discussion

Analysing how replacing the Cook Strait ferry with Zeppelins is an extreme hypothetical, and hence it is impossible to account for all possible changes. For example, we have assumed that the flow of people and goods through the Cook Strait will remain the same after such a change. However, it may be that the historical reputation of Zeppelins may discourage some prospective travellers, while the shorter travel times or new location may entice others.

Furthermore, whether this is practical is beyond the scope of the question. It may be that there is no way to build the infrastructure required, and hence no amount of airships would be sufficient.

Our model is also only valid for the now and immediate future. In reality, the conditions would have significantly changed by the time that a Zeppelin ferry service would be fully operational. Therefore, our model could be made more robust by considering changes to

[14] https://zeppelinflug.de/en/faqs

ferry demand or Zeppelin technology in the future.

However, it is near impossible to model how such things may evolve even in the near future.

Overall, our report has shown that there is a viable number of airships that can replace the Cook Strait ferries within the bounds of our reasonable assumptions.